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Mathematical and Analytical Support for the Functioning of the Anti-Crisis Management System in the Context of Ensuring Economic Security

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The article develops mathematical and analytical support for the functioning of the anti-crisis management system, focused on maintaining the economic security of the enterprise under the simultaneous action of financial, operational, market, and credit shocks. The proposed approach combines the normalization of indicators, entropy determination of local weights, the analytic hierarchy process for coordinating the weights of functional components, the multiplicative index of economic security, a dynamic model of the state of the enterprise, and optimization of the structure of anti-crisis actions using the Conditional Value-at-Risk criterion. The scientific idea is to transition from a static assessment of the financial condition to a controlled trajectory of economic security, for which the inertia of the internal state, the mutual influence of functional components, crisis load, limited anti-crisis budget, and the risk of adverse scenarios are taken into account. For testing, a transparent model dataset representing 10 hypothetical industrial enterprises and 16 indicators was formed. 5 000 scenario runs were performed with a reproducible random disturbance generator. The results confirmed that a uniform distribution of the anti-crisis resource is not rational for enterprises with different vulnerability structures. Optimized solutions reduced CVaR and the probability of falling below the critical threshold and generally increased the expected final index of economic security, although the effect depended on the enterprise's initial vulnerability profile. It was found that an increase in the normalized intensity of the anti-crisis budget from 0.08 to 0.18 for an enterprise with an average initial state increases the expected index at the end of the horizon from 0.427 to 0.593, and CVaR decreases from 0.941 to 0.407. The practical value of the model lies in the formation of a quantitative basis for choosing priorities for liquidity, operational recovery, supply diversification, market adaptation, and analytical and coordination support.

Keywords: *anti-crisis management; economic security; dynamic model; entropy method; hierarchy analysis method; CVaR; robust optimization.*

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1. Introduction

The functioning of enterprises in crisis conditions is accompanied by a simultaneous deterioration in liquidity, an increase in the cost of resources, supply disruptions, a reduction in effective demand and limited access to external financing. A separate response to each negative factor does not form a sufficient basis for maintaining economic security, since liquidity decisions affect the production program, supply actions change costs, and market diversification requires additional working capital. The anti-crisis management system should be considered as a controlled dynamic system, within which

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resource decisions change the future state of the enterprise, and not only compensate for already recorded losses.

The classic direction of quantitative diagnostics of a crisis state has formed around the forecasting of insolvency. Beaver [1] argued that financial ratios are informative for early detection of failure, Altman [2] proposed a multivariate discriminant model, Ohlson [3] used a probabilistic logit model, and Zmijewski [4] addressed problems of sampling and estimation bias. Generalizations from Balcaen and Ooghe [5] and Sun et al. [6] show that bankruptcy models are useful for signaling, but do not determine the structure of management actions, the volume of anti-crisis resources and the sequence of restoration of functional components. Consequently, the forecast of bankruptcy risk is not identical to the mathematical support for the functioning of the crisis management system.

Merton's structural approach [7] relates default risk to asset values, leverage and volatility. Its significance for this study lies in the interpretation of a crisis state as a probabilistic result of the interaction of assets, liabilities and uncertainty. At the same time, for managerial applications, it is necessary to supplement the financial direction with operational, market and information-analytical components. Shannon's entropy approach [8] makes it possible to determine the weight of indicators in accordance with their information usefulness, and not only according to subjective assessments. Markowitz's risk-return vision [9] forms the basis for the allocation of a limited resource, taking into account the expected outcome and uncertainty.

Assessing adverse deviations requires a metric that characterizes not only the average loss but also extreme scenarios. Rockafellar and Uryasev [10] proposed the Conditional Value-at-Risk optimization representation, which makes tail risk suitable for inclusion in objective functions. The work of Ben-Tal and Nemirovski [11] and Bertsimas and Sim [12] demonstrated the possibility of formulating solutions robust to parameter uncertainty. Under such conditions, anti-crisis policies should be evaluated not by a single expected scenario, but by the distribution of possible trajectories, including combined shocks.

The issue of enterprise resilience is also being explored in studies of supply chains and organizational adaptation. Christopher and Peck [13] link resilience with readiness, visibility, and restructuring of relationships; Sheffi and Rice [14] distinguish between redundancy and flexibility; and Ponomarov and Holcomb [15] define chain resilience as the ability to prepare, respond, and recover. Bhamra et al. [16] emphasize the multi-layered nature of resilience. Williams et al. [17] combine research on crisis management and organizational resilience, Duchek [18] views it as a meta-capacity of anticipation, coping, and adaptation, Ortiz-de-Mandojana and Bansal [19] demonstrate the long-term benefits of resilient practices, and Ivanov and Dolgui [20] expand the vision of renewal.

Existing research provides a strong theoretical and methodological foundation; however, a methodological gap remains between crisis diagnosis and the selection of anti-crisis measures. A static index does not indicate how security will change over several periods. The probability of bankruptcy does not determine what share of the budget to allocate to liquidity, operational recovery, supply diversification, market actions, or analytical support. Scenario forecasting without optimization, in turn, does not provide the best solution under the established constraints. The aim of this study is to develop mathematical and analytical support for an anti-crisis management system that enables the assessment of the initial level of economic security, the modeling its dynamics under the influence of crisis disturbances, and the optimization of the structure of anti-crisis actions based on the criterion of minimizing expected and tail losses.

The scientific novelty lies in the construction of an integrated model in which entropy weights are used for local aggregation of indicators, the analytic hierarchy process determines the significance of functional components, a multiplicative index captures the weak-link effect, a dynamic equation describes the state trajectory, and CVaR optimization determines the structure. This approach transforms economic security from a final characteristic into a manageable outcome.

2. Methodology

The research methodology consists of six sequential stages. Initially, a matrix of initial indicators is formed. Next, standardization is performed and entropy scales are calculated. In the third stage, the significance of functional components is coordinated using the analytic hierarchy process. In the fourth stage, the initial economic security index and the probability of a crisis are determined. In the fifth stage, a dynamic model of internal state change is constructed. The final stage involves optimizing the structure of anti-crisis measures and scenario testing of the results.

Let $i = 1, \dots, n$ denote the enterprise, $j = 1, \dots, m$ denote the indicator, $g \in \{F, O, M, C, H\}$ denote the functional component, and $t = 0, \dots, T$ denote the modeling period. The primary data matrix is represented as follows

$$X = [x_{ij}]_{n\times m}.$$

For stimulants, min–max normalization is applied

$$z_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}.$$

For destimulants, an inverse transformation is used

$$z_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}.$$

To prevent zero values in subsequent multiplicative aggregation, the normalized assessments of the functional components are stabilized using the parameter $\varepsilon = 0.05$

$$\tilde{z}_{ij} = \varepsilon + (1 - \varepsilon)\, z_{ij}.$$

Local indicator weights are determined using the entropy weighting method. The share of the i-th enterprise for the j-th indicator is calculated as follows

$$p_{ij} = \frac{z_{ij}}{\sum_{i=1}^{n} z_{ij}}.$$

The entropy of the indicator is expressed as follows

$$e_j = -\frac{1}{\ln n} \sum_{i=1}^{n} p_{ij} \ln p_{ij}.$$

Information utility and the local weight are determined using the following formulas

$$d_j = 1 - e_j, \quad w_j = \frac{d_j}{\sum_{j=1}^{m} d_j}.$$

The integrated assessment of each functional component is calculated as a weighted sum

$$I_{ig} = \sum_{j \in J_g} w_j \tilde{z}_{ij}, \quad g \in \{F, O, M, C\}.$$

Crisis pressure is constructed without an inverse transformation because an increase in its value has a negative interpretation

$$H_i = \sum_{j \in J_H} w_j^H h_{ij}.$$

To determine the weights of the functional components, the pairwise comparison matrix $A^{AHP} = [a_{kl}]$ is used. The priority vector $\boldsymbol{\omega}$ is obtained from the principal eigenvalue equation

$$A^{AHP}\boldsymbol{\omega} = \lambda_{\max}\, \boldsymbol{\omega}, \quad \sum_{g=1}^{4} \omega_g = 1.$$

The consistency of the matrix is verified using the following measures

$$\mathrm{CI} = \frac{\lambda_{\max} - q}{q - 1}, \quad \mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}_q},$$

where $q = 4$ and $\mathrm{RI}_4 = 0.90$. A value of $\mathrm{CR} < 0.10$ is considered acceptable.

The baseline economic security index is determined using a weighted geometric mean

$$B_i = \prod_{g\in\{F,O,M,C\}} I_{ig}^{\omega_g}.$$

The multiplicative form is used because a critically weak component should constrain the overall result. Arithmetic aggregation may conceal a liquidity deficit through a high assessment of managerial capacity, whereas the geometric approach prevents full compensation.

The adjusted economic security index is determined with allowance for crisis pressure

$$S_i^0 = B_i \exp\left(-\theta H_i\right),$$

where $\theta = 0.45$ reflects the sensitivity of the internal state to external and internal crisis factors. The probability of transition to a crisis state is specified by a logistic function

$$p_i^0 = \frac{1}{1+\exp\left[-\left(a - b\, S_i^0 + c\, H_i\right)\right]},$$

where, in the model validation, $a = 3.0$, $b = 6.0$, and $c = 1.8$.

The dynamic component of the model describes changes in the state vector $\boldsymbol{x}_{it} = (F_{it}, O_{it}, M_{it}, C_{it})^\top$

$$\boldsymbol{x}_{i,t+1} = \Pi_{[0,1]^4}\left(A\boldsymbol{x}_{it} + b_i B\boldsymbol{u}_i - G\boldsymbol{z}_{it} + \boldsymbol{\varepsilon}_{it}\right).$$

Matrix A captures inertia and the interdependence among the components

$$A = \begin{bmatrix} 0.965 & 0.010 & 0.005 & 0.015 \\ 0.010 & 0.960 & 0.015 & 0.015 \\ 0.010 & 0.015 & 0.960 & 0.015 \\ 0.015 & 0.015 & 0.010 & 0.965 \end{bmatrix}.$$

The control vector $\boldsymbol{u}_i = (u_L, u_O, u_S, u_M, u_A)^\top$ contains the shares of the anti-crisis budget allocated to liquidity support, operational recovery, supply diversification, market adaptation, and analytical and coordination support. The action-effectiveness matrix is given by

$$B = \begin{bmatrix} 0.70 & 0.12 & 0.04 & 0.08 & 0.16 \\ 0.10 & 0.62 & 0.62 & 0.08 & 0.18 \\ 0.04 & 0.08 & 0.30 & 0.70 & 0.18 \\ 0.16 & 0.16 & 0.10 & 0.16 & 0.72 \end{bmatrix}.$$

The crisis-factor impact matrix $\boldsymbol{z}_{it} = (z_D, z_C, z_S, z_K)^\top$ is given as follows

$$G = 0.58 \begin{bmatrix} 0.18 & 0.22 & 0.08 & 0.28 \\ 0.05 & 0.25 & 0.30 & 0.06 \\ 0.30 & 0.08 & 0.12 & 0.08 \\ 0.05 & 0.05 & 0.05 & 0.10 \end{bmatrix}.$$

The operator $\Pi_{[0,1]^4}$ constrains each state coordinate to the admissible interval. The budget constraint is formulated as follows

$$\sum_{k=1}^{5} u_{ik} = 1, \quad 0.02 \leqslant u_{ik} \leqslant 0.55.$$

For each scenario trajectory r, the adjusted index in period t is calculated using the formula

$$S_{itr} = \left(\prod_{g=1}^{4} x_{igtr}^{\omega_g}\right) \exp\left(-\vartheta\, \boldsymbol{q}^\top \boldsymbol{z}_{itr}\right),$$

where $\vartheta = 0.28$ and $\boldsymbol{q} = (0.28, 0.24, 0.28, 0.20)^\top$.

The trajectory loss function encompasses the shortfall of the overall index, shortfalls of individual components, and failure to recover by the end of the horizon

$$L_r(\boldsymbol{u}) = \sum_{t=1}^{T} \beta^t \left[\left(S^* - S_{itr}\right)_+^2 + \chi \sum_{g=1}^{4} \left(x_g^* - x_{igtr}\right)_+^2\right] + \psi\left(S^* - \min_t S_{itr}\right)_+ + \varphi\left(S^* - S_{iTr}\right)_+.$$

The calculations use $S^* = 0.58$, $\beta = 0.92$, $\chi = 0.22$, $\psi = 0.55$, and $\varphi = 0.35$. Value-at-Risk is defined as a quantile of the loss distribution

$$\mathrm{VaR}_\alpha(L) = \inf\left\{\ell \in \mathbb{R} \mid \Pr(L \leqslant \ell) \geqslant \alpha\right\}.$$

Conditional Value-at-Risk is expressed as follows

$$\mathrm{CVaR}_\alpha(L) = \mathbb{E}\left[L \mid L \geqslant \mathrm{VaR}_\alpha(L)\right].$$

For a discrete sample of N scenarios, the Rockafellar–Uryasev optimization representation is used

$$\widehat{\mathrm{CVaR}}_\alpha = \min_{\eta\in\mathbb{R}}\left[\eta + \frac{1}{(1-\alpha)N}\sum_{r=1}^{N}(L_r - \eta)_+\right].$$

The objective function of the anti-crisis management system is expressed as follows

$$\min_{\boldsymbol{u},\eta} J = \frac{1}{N}\sum_{r=1}^{N} L_r(\boldsymbol{u}) + \lambda\left[\eta + \frac{1}{(1-\alpha)N}\sum_{r=1}^{N}\left(L_r(\boldsymbol{u}) - \eta\right)_+\right] + \rho\,\|\boldsymbol{u}\|_2^2.$$

Parameter λ captures aversion to tail risk, whereas ρ limits excessive resource concentration. The main calculation uses $\alpha = 0.90$, $\lambda = 1.5$, $\rho = 0.05$, and $T = 6$. The training sample comprised 450 scenarios, and the results were validated using 5 000 independent scenarios. The random number generator seed was fixed, ensuring reproducibility of the calculations.

Table 1. Indicator system of the mathematical and analytical model.

Code	Economic interpretation	Type	Component
F1	Coverage of current liabilities with liquid assets	Stimulant	Financial resilience
F2	Share of equity in asset financing	Stimulant	Financial resilience
F3	Ability of operating profit to cover interest expenses	Stimulant	Financial resilience
O1	Capacity utilization without critical overloading	Stimulant	Operational resilience
O2	Diversification of suppliers of critical resources	Stimulant	Operational resilience
O3	Inverse indicator of downtime and emergency shutdowns	Stimulant	Operational resilience
M1	Diversification of revenue sources	Stimulant	Market adaptability
M2	Share of stable contracts in the order portfolio	Stimulant	Market adaptability
M3	Inverse indicator of customer-base concentration	Stimulant	Market adaptability
C1	Speed of preparing and adopting anti-crisis decisions	Stimulant	Managerial capacity
C2	Readiness of scenario plans and contingency procedures	Stimulant	Managerial capacity
C3	Completeness of information support for managerial decision-making	Stimulant	Managerial capacity
H1	Decline in demand in key markets	Destimulant	Crisis pressure
H2	Increase in resource costs and production expenses	Destimulant	Crisis pressure
H3	Disruptions to supply and logistics links	Destimulant	Crisis pressure
H4	Restricted access to credit and working capital	Destimulant	Crisis pressure

Table 2. Initial assessments of financial and operational resilience.

	F1	F2	F3	O1	O2	O3
E1	82.000	76.000	78.000	81.000	74.000	79.000
E2	88.000	84.000	86.000	70.000	66.000	73.000
E3	69.000	64.000	67.000	72.000	70.000	68.000
E4	55.000	48.000	52.000	60.000	56.000	54.000
E5	63.000	57.000	60.000	52.000	49.000	50.000
E6	71.000	66.000	69.000	65.000	62.000	60.000
E7	75.000	70.000	72.000	86.000	82.000	84.000
E8	48.000	43.000	46.000	47.000	45.000	42.000
E9	77.000	72.000	75.000	69.000	68.000	71.000
E10	66.000	61.000	64.000	67.000	64.000	66.000

The input dataset is a reference model set. It is formed on a 0–100 scale to test the mathematical consistency, sensitivity, and managerial interpretation of the model. The dataset does not reflect the financial statements of specific enterprises and is not used to make statements regarding their actual performance. This limitation is essential, as methodological validation must be separated from the empirical identification of a specific economic entity.

Table 3. Initial assessments of market adaptability and managerial capacity.

	M1	M2	M3	C1	C2	C3
E1	72.000	78.000	70.000	80.000	77.000	82.000
E2	68.000	75.000	64.000	76.000	72.000	78.000
E3	65.000	69.000	61.000	67.000	64.000	70.000
E4	49.000	52.000	46.000	58.000	51.000	55.000
E5	58.000	61.000	54.000	64.000	59.000	62.000
E6	54.000	58.000	50.000	82.000	80.000	84.000
E7	63.000	66.000	60.000	71.000	68.000	73.000
E8	44.000	48.000	41.000	50.000	46.000	49.000
E9	84.000	82.000	80.000	79.000	75.000	81.000
E10	60.000	64.000	57.000	69.000	66.000	71.000

Table 4. Initial assessments of crisis pressure.

	H1	H2	H3	H4
E1	28.000	35.000	30.000	25.000
E2	32.000	42.000	38.000	29.000
E3	45.000	52.000	47.000	40.000
E4	68.000	72.000	74.000	63.000
E5	61.000	69.000	78.000	58.000
E6	49.000	57.000	55.000	44.000
E7	38.000	46.000	34.000	36.000
E8	80.000	77.000	82.000	75.000
E9	36.000	48.000	43.000	31.000
E10	52.000	59.000	51.000	47.000

3. Results

Entropy weighting revealed that within the financial component, financing autonomy has the greatest informational utility, with a weight of 0.349. For the operational component, supplier diversification has the highest weight, 0.345. Within the market component, the inverse indicator of customer-base concentration is slightly dominant, at 0.339. For management capability, the most important factors were the readiness of scenario plans and the completeness of information support. Within the crisis burden, supply disruptions and credit restrictions received the highest weights. This result is consistent with the nature of crisis management, as interruptions in material flows and working capital shortages often transform a local problem into a systemic crisis.

Table 5. Entropy weights of indicators.

Indicator	Entropy weight
F1	0.320
F2	0.349
F3	0.331
O1	0.334
O2	0.345
O3	0.320
M1	0.330
M2	0.331
M3	0.339
C1	0.308
C2	0.351
C3	0.341
H1	0.263
H2	0.195
H3	0.271
H4	0.271

The analytic hierarchy process determined the weights of the functional components at 0.317 for financial stability, 0.235 for operational stability, 0.189 for market adaptability, and 0.259 for managerial capability. The matrix's principal eigenvalue is $\lambda_{\max} = 4.00035$, the consistency index $\mathrm{CI} = 0.000116$, and the consistency ratio $\mathrm{CR} = 0.000128$. The value is significantly lower than the 0.10 limit, so the matrix does not contain contradictory statements. The dominance of the financial component does not imply the secondary importance of the operational or market components. The multiplicative index ensures a strong reaction to any critically weak coordinate.

Initial integrated results revealed four different status types. Enterprises E1, E2, and E9 have a high base index and relatively low crisis burden. E7 demonstrates a strong operating component, but market adaptability lowers the overall result. E3, E6, and E10 belong to an intermediate group, for which anti-crisis measures can significantly change the trajectory. E4, E5, and E8 are characterized by critical structural weakness, so short-term optimization of resource allocation is insufficient without recapitalization, changes in production configuration, or restructuring of liabilities.

Table 6. Pairwise comparison matrix and component weights.

	F	O	M	C	Weight
F	1.000	1.350	1.700	1.200	0.317
O	0.741	1.000	1.250	0.900	0.235
M	0.588	0.800	1.000	0.750	0.189
C	0.833	1.111	1.333	1.000	0.259

Table 7. Initial integrated assessments of economic security.

	F	O	M	C	H	B	S_0	p_0
E1	0.827	0.852	0.786	0.934	0.000	0.851	0.851	0.108
E2	1.000	0.648	0.678	0.811	0.116	0.795	0.754	0.211
E3	0.545	0.664	0.574	0.576	0.335	0.585	0.503	0.642
E4	0.191	0.340	0.167	0.228	0.809	0.223	0.155	0.971
E5	0.387	0.184	0.387	0.426	0.754	0.333	0.237	0.949
E6	0.592	0.478	0.295	1.000	0.442	0.565	0.463	0.734
E7	0.678	1.000	0.522	0.680	0.182	0.707	0.652	0.358
E8	0.050	0.050	0.050	0.050	1.000	0.050	0.032	0.990
E9	0.733	0.643	1.000	0.896	0.201	0.794	0.725	0.271
E10	0.474	0.555	0.456	0.624	0.461	0.524	0.426	0.782

Optimization of the anti-crisis resource structure revealed the absence of a universal correlation between actions. For E1, which has a balanced initial state, the distribution remains close to uniform. For E3, E7, E9, and E10, liquidity maintenance predominates. E5 requires the resource to be primarily directed toward operational recovery and supply diversification. For E4 and E8, the optimizer concentrates resources on liquidity or analytical and coordination support, but the final index remains low. This result is an important management signal, as the mathematical model does not create the artificial illusion of recovery where the initial deficit is too significant.

Table 8. Optimized structure of anti-crisis actions and scenario validation results.

	uL	uO	uS	uM	uA	S eq.	S opt.	CVaR eq.	CVaR opt.
E1	0.263	0.187	0.176	0.181	0.194	0.806	0.810	0.001	0.000
E2	0.202	0.191	0.296	0.071	0.241	0.757	0.761	0.007	0.004
E3	0.550	0.020	0.214	0.020	0.196	0.521	0.543	0.475	0.392
E4	0.390	0.020	0.020	0.020	0.550	0.140	0.152	2.600	2.550
E5	0.020	0.385	0.550	0.020	0.025	0.241	0.229	2.040	1.984
E6	0.305	0.020	0.407	0.248	0.020	0.502	0.512	0.692	0.644
E7	0.524	0.020	0.020	0.288	0.148	0.665	0.684	0.132	0.089
E8	0.020	0.020	0.020	0.390	0.550	0.049	0.060	3.193	3.046
E9	0.508	0.213	0.176	0.020	0.083	0.737	0.758	0.044	0.013
E10	0.550	0.020	0.326	0.084	0.020	0.444	0.463	0.838	0.724

For E3, optimization increased the expected final index from 0.521 to 0.543 and decreased the CVaR from 0.475 to 0.392. For E7, the final index increased from 0.665 to 0.684, and tail loss decreased from 0.132 to 0.089. For E9, the result increased from 0.737 to 0.758, while the CVaR decreased from 0.044 to 0.013. The most practically significant effect was obtained for E10, where the optimized solution increased the final index from 0.444 to 0.463 and decreased tail loss from 0.838 to 0.724. The probability of moving below the 0.45 level decreased from 0.530 to 0.446. For a detailed scenario analysis, E6 was selected, which has an average financial position, moderate operational stability, weaker market adaptability, and high management capacity. The optimal solution assumes 30.5% of the resource allocated to liquidity, 40.7% to supply diversification, and 24.8% to market adaptability. Minimal shares are allocated to operational recovery and analytical and coordination support, as the initial management component is already strong, and supply diversification simultaneously supports operational and market coordinates.

The optimistic scenario sees the index rise from 0.556 to 0.667. The baseline trajectory reaches 0.577 and is practically approaching the target level of 0.58. A price shock reduces the final result to 0.499, while a supply disruption reduces it to 0.472. The combined crisis causes a decline to 0.346. Consequently, resource optimization increases resilience but does not eliminate the consequences of a prolonged, simultaneous shock to demand, expenditures, supply, and credit. Under these conditions, the company requires not only internal redistribution decisions but also external financing, a review of the debt structure, and a change in the production and market configuration.

Table 9. Scenario dynamics of the economic security index for Enterprise E6.

Period	Optimistic	Baseline	Price shock	Supply disruption	Combined crisis
0.000	0.556	0.550	0.545	0.542	0.534
1.000	0.577	0.556	0.539	0.531	0.504
2.000	0.597	0.561	0.532	0.520	0.474
3.000	0.615	0.566	0.525	0.509	0.443
4.000	0.633	0.570	0.517	0.497	0.411
5.000	0.651	0.573	0.508	0.484	0.379
6.000	0.667	0.577	0.499	0.472	0.346

A sensitivity analysis of the anti-crisis budget intensity revealed a nonlinear effect. At $b = 0.08$, the expected final index is 0.427, and the probability of falling below 0.45 is 0.552. Increasing the parameter to 0.13 increases the index to 0.512 and reduces the indicated probability to 0.162. A further increase to 0.18 yields an expected index of 0.593 and a CVaR of 0.407. However, the critical state probability decreases only to 0.146, since extreme combined shocks cannot be fully offset by the domestic budget.

Table 10. Sensitivity of Enterprise E6 results to anti-crisis budget intensity.

b	uL	uO	uS	uM	uA	$E(S_6)$	$Q_{0\cdot 1}(S_6)$	$CVaR_{0\cdot 9}$	$P(S_6 < 0.45)$
0.080	0.252	0.020	0.416	0.292	0.020	0.427	0.216	0.941	0.552
0.130	0.305	0.020	0.407	0.248	0.020	0.512	0.309	0.644	0.162
0.180	0.326	0.020	0.393	0.241	0.020	0.593	0.396	0.407	0.146

The calculations provide grounds for the formation of three management response zones. For $S_i^0 \geqslant 0.65$ and $H_i \leqslant 0.25$, a preventive policy with reserve maintenance and controlled diversification is advisable. For $0.40 \leqslant S_i^0 < 0.65$, optimization of the anti-crisis budget structure and periodic scenario updates are necessary. For $S_i^0 < 0.40$, internal resource redistribution should be supplemented by recapitalization, debt restructuring, reduction of fixed costs, or the withdrawal of some assets from non-core use.

4. Discussions

The obtained results expand on the classical approach to forecasting financial insolvency. The models of Beaver [1], Altman [2], Ohlson [3], and Zmijewski [4] answer questions about the probability of deterioration or bankruptcy, while the proposed model determines how to allocate anti-crisis resources to change the future trajectory. Thus, the forecast signal turns into an optimization problem. At the same time, financial indicators remain the central component, as evidenced by the highest AHP weight for financial stability.

Multiplicative aggregation is consistent with the findings of organizational resilience research. Duchek [18] considers resilience as a combination of prediction, coping, and adaptation, while Williams et al. [17] and others [22–24] emphasize the interrelationship between preparation, response, and recovery. A high score on a single component cannot ensure the desired level of security. Result E6 confirms this position, since maximum managerial capacity does not compensate for weaker market adaptability and moderate financial headroom. The structure of optimal solutions is consistent with the studies of Christopher and Peck [13], Sheffi and Rice [14], Ponomarov and Holcomb [15], and Ivanov and Dolgui [20]. Supply diversification is not solely a logistical action. In the proposed performance matrix, it influences the operational and market components, and through intercomponent links, it influences financial performance. For E5 and E6, this area received the largest share of resources. Thus, the mathematical model reflects the network nature of the company's economic security.

The use of CVaR made it possible to avoid a solution focused solely on the average scenario. A company can demonstrate an acceptable expected value and still incur unacceptable losses in extreme cases. The approach of Rockafellar and Uryasev [10] provides a quantitative representation of this risk. The robust approach of Ben-Tal and Nemirovski [11] and Bertsimas and Sim [12] is reflected through scenario selection, resource concentration constraints, and the inclusion of a tail component in the objective function.

In our opinion, the key advantage of the proposed approach lies in the distinction between three management situations. The first situation implies sufficient internal reserves, when optimization provides a primarily preventive effect. The second situation is characterized by recurring vulnerability, in which the budget structure significantly alters risk. The third situation reflects structural insolvency, when internal anti-crisis measures are unable to return the enterprise to an acceptable level without external intervention. Therefore, the model performs not only a distribution function but also a diagnostic one.

Practical use requires adapting the parameters to a specific enterprise. Matrices A, B, and G should be estimated using time series, the results of management experiments, or an expert survey. The target level of S^* should be consistent with industry specifics, the duration of the production cycle, the debt structure, and the acceptable level of safety. This parameter λ is determined by the degree of aversion to extreme losses for owners and creditors.

The study has limitations. The validation was performed on a model array, so the obtained numerical values cannot be transferred to specific enterprises without re-identification. The dynamic equation is linear for the projection operator, although actual responses may contain threshold and delayed effects. The scenarios simulate four groups of shocks and do not account for personnel losses, technological accidents, regulatory restrictions, or reputational events. Further research should focus on constructing a panel estimation of transition matrices, incorporating network dependencies between enterprises, and using a variable multi-period control policy $\boldsymbol{u}_t$ depending on the actual state.

5. Conclusions

The study developed a mathematical and analytical framework for the functioning of an anti-crisis management system in the context of economic security. The proposed model combines four internal components, four crisis-pressure groups, and five areas of anti-crisis action. Entropy weighting ensures the objectification of local weights, the analytic hierarchy process aligns the importance of the components, and the multiplicative index takes into account the weakest link effect.

The dynamic model formalizes the inertia of the state, the mutual influence of financial, operational, market, and management components, the impact of external and internal shocks, and the effectiveness of the anti-crisis resource. The CVaR criterion allows for the assessment of not only expected but also extreme losses. A simulation using 5 000 test scenarios confirmed that the optimal action structure depends on the initial vulnerability profile of the enterprise.

The results showed that for enterprises with a recoverable state, optimization increases the final economic security index and reduces tail losses. For companies with critically low initial estimates, internal budget reallocation does not provide sufficient recovery. Consequently, the model allows for a timely distinction between situations requiring optimization of current actions and those requiring recapitalization, debt restructuring, or operational reconfiguration.

A sensitivity analysis confirmed the significance of the intensity of the anti-crisis budget. For model E6, increasing the parameter from 0.08 to 0.18 increased the expected final index from 0.427 to 0.593 and decreased the CVaR from 0.941 to 0.407. At the same time, extreme combined shocks maintain the risk of a critical state, demonstrating the need for a combination of internal and external instruments to maintain economic security.

The proposed mathematical and analytical model can be used at companies for quarterly or monthly revisions of anti-crisis budget priorities, scenario testing of management decisions, and the generation of early signals of structural insolvency. Further empirical testing on real panel data will allow for the evaluation of transition matrices and the effectiveness of actions for individual industries.

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Математико-аналітичне забезпечення функціонування системи антикризового менеджменту в контексті уможливлення економічної безпеки

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У статті розроблено математико-аналітичне забезпечення функціонування системи антикризового менеджменту, зорієнтоване на підтримання економічної безпеки підприємства за одночасної дії фінансових, операційних, ринкових і кредитних шоків. Запропонований підхід поєднує нормування показників, ентропійне визначення локальних ваг, метод аналізу ієрархій для узгодження ваг функціональних складових, мультиплікативний індекс економічної безпеки, динамічну модель стану підприємства та оптимізацію структури антикризових дій за критерієм Conditional Value-at-Risk. Наукова ідея полягає у переході від статичної оцінки фінансового стану до керованої траєкторії економічної безпеки, для якої враховано інерцію внутрішнього стану, взаємний вплив функціональних складових, кризове навантаження, обмеженість антикризового бюджету та ризик несприятливих сценаріїв. Для апробації сформовано прозорий модельний масив із 10 умовних промислових підприємств і 16 показників. Виконано 5000 сценарних прогонів із відтворюваним генератором випадкових збурень. Результати підтвердили, що рівномірний розподіл антикризового ресурсу не є раціональним для підприємств із різною структурою вразливості. Оптимізовані рішення скоротили хвостові втрати CVaR, підвищили очікуваний кінцевий індекс економічної безпеки та зменшили ймовірність переходу нижче критичної межі. Встановлено, що збільшення нормованої інтенсивності антикризового бюджету з 0.08 до 0.18 для підприємства із середнім початковим станом підвищує очікуваний індекс у кінці горизонту з 0.427 до 0.593, а CVaR знижується з 0.941 до 0.407. Практичне значення моделі полягає у формуванні кількісного підґрунтя для вибору пріоритетів ліквідності, операційного відновлення, диверсифікації постачання, ринкової адаптації та аналітико-координаційного забезпечення.

Ключові слова: *антикризовий менеджмент; економічна безпека; динамічна модель; ентропійний метод; метод аналізу ієрархій; CVaR; робастна оптимізація.*