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XX МІЖНАРОДНА НАУКОВА КОНФЕРЕНЦІЯ ІМЕНІ АКАДЕМІКА МИХАЙЛА КРАВЧУКА

17–20 листопада 2025 р.

Національний технічний університет України
«Київський політехнічний інститут імені Ігоря Сікорського»
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A LIMIT THEOREM FOR STOCHASTIC EQUATIONS WITH THE LOCAL TIME

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The stochastic differential equation that involves the local time of the process was first investigated in [1] and [2]. Moreover, in works [2], [3], and [4], formulae were obtained that connect solutions of stochastic equations with local time to solutions of Itô's stochastic equations.

It is well known that the convergence of coefficients in an Itô stochastic equation is recognized as insufficient for guaranteeing the weak convergence of the equation's solutions; an additional condition is required.

We consider solutions of stochastic equations that involve local time, focusing on cases where the coefficients exhibit nonregular dependence on a small parameter ε:

$$\xi_\varepsilon(t) = x+\beta_\varepsilon L^{\xi_\varepsilon}(t,0)+\int_0^t \big(b_\varepsilon(\xi_\varepsilon(s))+g_\varepsilon(\xi_\varepsilon(s))\big)ds+\int_0^t \sigma_\varepsilon(\xi_\varepsilon(s))dw(s). \quad (1)$$

The primary focus is on studying the weak convergence of the solutions of these equations (specifically, in terms of the weak convergence of the measures generated by the processes in a certain functional space) as $\varepsilon \to 0$.

Denote by $\xi(t)$ a weak solution following a stochastic equation involving a local time (with $|\gamma| < 1$)

$$\xi(t) = x + \gamma L^{\xi}(t,0) + \int\limits_0^t g(\xi(s))ds + \int\limits_0^t \sigma(\xi(s))dw(s). \quad (2)$$

Let $(\mathbb{C}[0,T], C_t), t \in [0,T]$ be a space of continuous functions in the interval $[0,T]$. Consider the measures on the functional space $(\mathbb{C}[0,T], C_t)$ generated by the processes $\xi_\varepsilon(t)$ and $\xi(t)$, respectively.

We suppose that the coefficients of the stochastic equations (1) and (2) satisfy the following conditions: $\beta_\varepsilon \to \beta$ when $\varepsilon \to 0$, $|\beta_\varepsilon| < 1$ and $|\beta| < 1$, there exists a constant $\Lambda > 0$ such that $|g_\varepsilon(x)| \leqslant \Lambda$, $|g(x)| \leqslant \Lambda$, $\dfrac{1}{\Lambda} \leqslant \sigma_\varepsilon^2(x) \leqslant$

This work was partially supported by a grant from the Simons Foundation (SFI-PD-Ukraine-00017674, Krykun I. H.).

Λ, $\dfrac{1}{\Lambda} \leqslant \sigma^2(x) \leqslant \Lambda$ and for every $x \in \mathbb{R}$

$$\left| \int_0^x \frac{b_\varepsilon(y)}{\sigma_\varepsilon^2(y)} dy \right| \leqslant \Lambda \,.$$

Based on the research detailed in the paper [5], the following result has been achieved.

Theorem 1. *The necessary and sufficient conditions are obtained for the weak convergence of solutions of stochastic equations (1) to the solution of stochastic equation (2).*

The detailed formulation of the necessary and sufficient conditions for convergence, along with the proof of this theorem and supporting results, is presented in a manuscript currently undergoing peer review.

Acknowledgments. *The Author expresses his heartfelt gratitude to the brave soldiers of the Ukrainian Armed Forces who have protected the lives of the author, his family, and his friends from Russian bloody murderers since 2014.*

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