On the Weak Convergence of Stochastic Equations Involving Local Time and Non-Regular Coefficients

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It is well known that the convergence of coefficients in an Itô's stochastic equation is insufficient for the weak convergence of its solutions; an additional condition is required.

This paper investigates the weak convergence of solutions to stochastic equations involving local time, specifically addressing cases with non-regular dependence of coefficients on a small parameter ε:

$$\xi_\varepsilon(t) = x + \beta_\varepsilon L^{\xi_\varepsilon}(t,0) + \int_0^t \big(b_\varepsilon(\xi_\varepsilon(s)) + g_\varepsilon(\xi_\varepsilon(s))\big)ds + \int_0^t \sigma_\varepsilon(\xi_\varepsilon(s))dw(s). \quad (1)$$

A stochastic equation involving local time of a process was first investigated in [1] and [2]. Moreover in [2], [3], [4] were obtained formulae which connect solutions of stochastic equations with local time of a process with solutions of Itô's stochastic equations.

We assume that the coefficients of stochastic equation (1) satisfy the following conditions: $\beta_\varepsilon \to \beta$ when $\varepsilon \to 0$, $|\beta_\varepsilon| < 1$ and $|\beta| < 1$, there exists a constant $\Lambda > 0$ such that $|g_\varepsilon(x)| < \Lambda$, $\dfrac{1}{\Lambda} < \sigma_\varepsilon^2(x) < \Lambda$ and for every $x \in \mathbb{R}$

$$\left| \int_0^x \frac{b_\varepsilon(y)}{\sigma_\varepsilon^2(y)} dy \right| \le \Lambda\,.$$

Denote by $\xi(t)$ a weak solution following stochastic equation involving a local time of a process (with $|\gamma| < 1$)

$$\xi(t) = x + \gamma L^{\xi}(t,0) + \int_0^t g(\xi(s))ds + \int_0^t \sigma(\xi(s))dw(s). \tag{2}$$

Let $(\mathbb{C}[0,T], C_t), t \in [0,T]$ be a space of continuous functions on the interval $[0,T]$. Let μ_ε and μ denote the measures on functional space $(\mathbb{C}[0,T], C_t)$ generated by the processes $\xi_\varepsilon(t)$ and $\xi(t)$, respectively.

Theorem 1. *We establish in [5, 6] the necessary and sufficient conditions for the weak convergence of solutions of stochastic equations (1) to the solution of stochastic equation (2).*

Acknowledgment. *This work was partially supported by a grant from the Simons Foundation (SFI-PD-Ukraine-00017674, Krykun I.H.).*

The author expresses his sincere gratitude to the heroic soldiers of the Armed Forces of Ukraine, who have been protecting the author, his family, and his friends from the bloody Russian terrorist forces since 2014.

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